In-plane motion measurements with
Fourier lensless holography

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A holographic and moiré technique for measuring in­
plane motions of a holographic plate attached to an object is
described. This one seems easier to implement than a similar
approach given by Roychoudhuri and Machorro. 1

It is well known that when spherical and plane coherent
wave fronts interfere, they give rise to a Fresnel-zonelike in­
terference pattern. This is so when a lensless Fourier holo­
gram of an empty pupil is recorded. 2 If between two such
exposures the holographic plate is rotated or translated in­
plane, the two overlapping interference patterns will produce
low-frequency moiré patterns. If the displacement involved
is small (less than the innermost Fresnel ring), the
moire pattern consists of equally spaced straight lines perpendic­ular to the direction of the displacement, 3 and the interfringe
spacing is inversely proportional to the magnitude of the
displacement.

In this case, due to the symmetry of the interference fringes,
The spherical reference wave has been approximated by a parabolical surface, as is usual in Fresnel approximation.\textsuperscript{4}

If the photographic plate rotates an angle $\theta$ between exposures, the second one can be described by

$$I_2 = A_1^2 + A_2^2 + 2A_1A_2 \cos \left\{ \frac{\pi}{\lambda_2} [x'(x_0 \cos \theta + y_0 \sin \theta - x_0) + y(y_0 \cos \theta - x_0 \sin \theta - y_0)] \right\} \cos \left\{ \frac{\pi}{\lambda_2} [x^2 + y^2 + x_0^2 + y_0^2 - x(x_0 \cos \theta + y_0 \sin \theta - x_0) - y(y_0 \cos \theta - x_0 \sin \theta - y_0)] \right\} \times (\sin \theta + y_0) + \phi,$$

where

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}.$$

Suppose that the registers are done in the linear region of the $(t - E)$ curve of the plate; the transmittance of the developed plate will be

$$t \alpha I_1 + I_2;$$

thus,

$$t \alpha 2A_1^2 + 2A_2^2 + 4A_1A_2 \cos \left\{ \frac{\pi}{\lambda_2} [x(x_0 \cos \theta + y_0 \sin \theta - x_0) + y(y_0 \cos \theta - x_0 \sin \theta - y_0)] \right\} \cos \left\{ \frac{\pi}{\lambda_2} [x^2 + y^2 + x_0^2 + y_0^2 - x(x_0 \cos \theta + y_0 \sin \theta - x_0) - y(y_0 \cos \theta - x_0 \sin \theta - y_0)] \right\} \times (\sin \theta + y_0) + \phi.$$

It can be seen that it consists of a high frequency family of curves modulated by a low frequency family of parallel lines represented by the first cosine. The last one is usually called the moiré pattern.

A straightforward calculation shows that the spacing of the moiré fringes is

$$\Delta = \frac{\lambda z}{2(x_0^2 + y_0^2)^{1/2} \sin \left( \frac{\theta}{2} \right)}, \quad (2)$$

that is, inversely proportional to the magnitude of the displacement of the center of the Fresnel zones. Also the slope of these fringes is

$$m = \frac{(y_0/x_0) - \tan \left( \frac{\theta}{2} \right)}{1 + (y_0/x_0) \tan \left( \frac{\theta}{2} \right)}; \quad (3)$$

that is, the fringes bisect the angle of rotation.

When the images of this hologram are observed in the Fourier plane, it can be seen that the mentioned moiré fringes act as a low frequency linear grating producing two tiny bright spots, located symmetrically to the zero order, the separation $d$ between which can be easily calculated from the given spacing $\Delta$, as

$$d = \frac{\lambda D}{\Delta} (x_0^2 + y_0^2)^{1/2} \sin \left( \frac{\theta}{2} \right);$$

where $D$ is the distance between the hologram and the Fourier plane when the former is illuminated with a spherical convergent beam. That is, the separation between the spots is proportional to the magnitude of the displacement of the center of the Fresnel zones.

Returning to expression (1), it may be written as follows:

...
\[ t \alpha (2A_1^2 + 2A_2^2) + A_1A_2 \left[ \exp(\mathrm{i}\phi) \right. \]

\[ \exp \left[ \frac{-i\pi}{\lambda z} [(x - x_0)^2 + (y - y_0)^2] \right] \]

\[ + \exp(\mathrm{i}\phi') \exp \left[ \frac{-i\pi}{\lambda z} [(x - x_0)^2 + (y - y_0)^2] \right] \]

\[ + A_1A_2 \left( \exp(-i\phi) \exp \left[ \frac{-i\pi}{\lambda z} [(x - x_0)^2 + (y - y_0)^2] \right] \right) \]

\[ + \exp(-i\phi') \exp \left[ \frac{-i\pi}{\lambda z} [(x - x_0)^2 + (y - y_0)^2] \right] \]

\[ + [(y - y_0 + \beta)^2] \right] \]

where \( \alpha = x_0(\cos \theta - 1) + y_0 \sin \theta \), \( \beta = y_0(\cos \theta - 1) - x_0 \sin \theta \), and \( \phi' \) is a constant. The first term will give rise to the zero order, and the other two terms, to the plus-one and minus-one orders, respectively. Taking into account only the plus-one order, it may be expressed as

\[ I^{(+1)\alpha} A_1A_2 \exp(\mathrm{i}\phi) \exp \left[ \frac{i\pi}{\lambda z} [(x - x_0)^2 + (y - y_0)^2] \right] \]

\[ \ast \delta(x) \delta(y) + \exp[i(\phi' - \phi)] \delta(x - \alpha) \delta(y - \beta), \]

where \( \ast \) denotes a convolution product.

The corresponding intensity in the Fourier plane will be

\[ I = C \left( 1 + \exp \left[ -i \frac{2\pi}{\lambda D} (x_0 + y_0) - (\phi' - \phi) \right] \right)^2 \]

\[ = 2C \left[ 1 + \cos \frac{2\pi}{\lambda D} (x_0 + y_0) - (\phi' - \phi) \right]. \]

that is, a family of straight lines. It is easily shown that the slope of this family is also given by expression (3). The spacing \( \Delta' \) between the straight lines results proportional to expression (2), that is,

\[ \Delta' = \frac{\lambda D}{2(x_0^2 + y_0^2)^{1/2} \sin \left( \frac{\theta}{2} \right)} = \frac{D \Delta}{Z}. \]

Neither the pupil in the object plane during the exposures nor the one limiting the reconstruction process was taken into account in calculations, but their inclusion does not affect the final results substantially.

The experimental setup is shown in Fig. 1. A part of a collimated laser beam is focused on the object plane by a small lens, while the rest of the beam passes through pupil \( P \). Both waves interfere on plate \( H \), which is rotated between exposures. Figures 2 and 3 show the moiré fringes in the hologram and the intensity in the Fourier plane. Results confirm the described method. If two spherical wave fronts are used instead of one, the location of the axis of rotation can be determined.

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