In-plane motion measurements with Fourier lensless holography

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A holographic and moiré technique for measuring in-plane motions of a holographic plate attached to an object is described. This one seems easier to implement than a similar approach given by Roychoudhuri and Machorro.[1]

It is well known that when spherical and plane coherent wave fronts interfere, they give rise to a Fresnel-zonelike interference pattern. This is so when a lensless Fourier hologram of an empty pupil is recorded.[2] If between two such exposures the holographic plate is rotated or translated in-plane, the two overlapping interference patterns will produce low-frequency moiré patterns. If the displacement involved is small (less than the innermost Fresnel ring), the moiré pattern consists of equally spaced straight lines perpendicular to the direction of the displacement,[3] and the interfringe spacing is inversely proportional to the magnitude of the displacement.

In this case, due to the symmetry of the interference fringes, translations and rotations give rise to the same type of moiré fringes. These moiré fringes can be useful as a primary tool to determine the magnitude and direction of the movement and the angle of rotation if the location of the axis is known. Besides, these moiré fringes give rise, in the reconstruction plane of the hologram (Fourier plane), to two bright spots separated by a distance proportional to the magnitude of the displacement or rotation. Another cooperative result consists of Young's fringes in the image of the reconstructed pupil, the interfringe of which is also inversely proportional to the magnitude of the movement.

A simple explanation can be given for the above observations. The intensity due to the first exposure in the holographic plate is given by

$$I_1 = A_1^2 + A_2^2 + 2A_1A_2 \times \cos\left\{\frac{\pi}{\lambda z}\left[(x - x_0)^2 + (y - y_0)^2\right] + \phi\right\},$$

where A_1 is the amplitude of the spherical wave, and A_2 is the amplitude of the plane wave in the photographic plate plane, λ is the wavelength of the monochromatic light, z is the distance between the object plane and the recording plane, (x,y) are the coordinates of an arbitrary point in the plate, (x_0,y_0) are the coordinates of the perpendicular projection of the reference point, and ϕ is a constant phase delay. It should be noted that the origin of the coordinate system is chosen at the point where the axis of rotation intersects the recording plane.

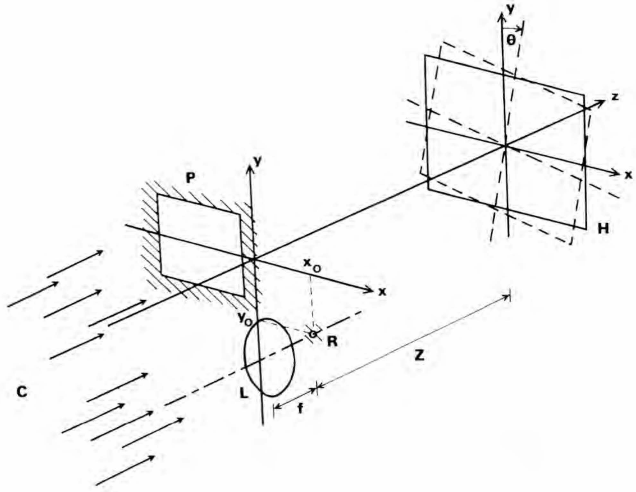

Fig. 1. Experimental setup. C, collimated beam of a He–Ne laser; P, pupil; L, lens of $f = +2$ cm focal length; R, reference spatial filter; and H, recording plate. The distance Z from the object plane to the plate was 206 cm.

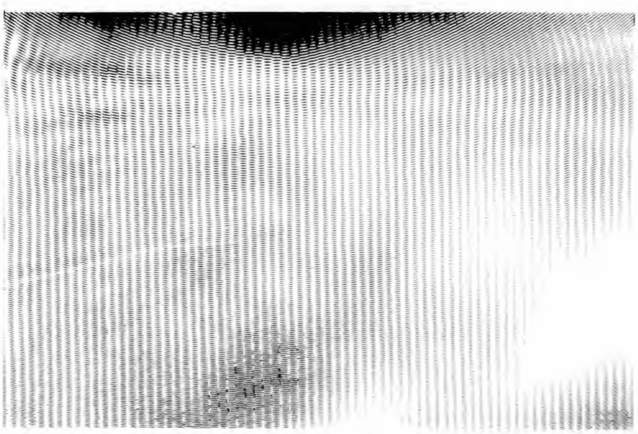

Fig. 2. Moiré fringes in a double-exposure hologram (enlarged image). The region of nonoverlapping of individual exposures is clearly observed in the upper part of the plate.

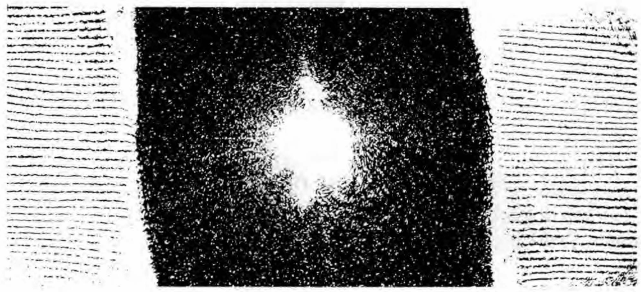

Fig. 3. Positive reproduction of the Fourier plane image. In the central region and close to the zero order, two symmetrical spots can be observed. The rotation corresponds to an angle $\theta = 10°$. At both sides Young's fringes modulate the reconstructed images.

The spherical reference wave has been approximated by a parabolical surface, as is usual in Fresnel approximation.[4]

If the photographic plate rotates an angle θ between exposures, the second one can be described by

$$I_2 = A_1^2 + A_2^2 + 2A_1A_2 \times \cos\left\{\frac{\pi}{\lambda z}\left[(x' - x_0)^2 + (y' - y_0)^2\right] + \phi\right\},$$

where

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & +\cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

Suppose that the registers are done in the linear region of the $(t - E)$ curve of the plate; the transmittance of the developed plate will be

$$t \alpha I_1 + I_2; \tag{1}$$

thus,

$$t \alpha 2A_1^2 + 2A_2^2 + 4A_1A_2 \cos\left\{\frac{\pi}{\lambda z}\left[x(x_0\cos\theta + y_0\sin\theta - x_0) + y(y_0\cos\theta - x_0\sin\theta - y_0)\right]\right\} \cos\left\{\frac{\pi}{\lambda z}\left[x^2 + y^2 + x_0^2 + y_0^2 - x(x_0\cos\theta + y_0\sin\theta + x_0) - y(y_0\cos\theta - x_0) \times (\sin\theta + y_0)\right] + \phi\right\}.$$

It can be seen that it consists of a high frequency family of curves modulated by a low frequency family of parallel lines represented by the first cosine. The last one is usually called the moiré pattern.

A straightforward calculation shows that the spacing of the moiré fringes is

$$\Delta = \frac{\lambda z}{2(x_0^2 + y_0^2)^{1/2}\sin\left(\frac{\theta}{2}\right)}, \tag{2}$$

that is, inversely proportional to the magnitude of the displacement of the center of the Fresnel zones. Also the slope of these fringes is

$$m = \frac{(y_0/x_0) - \tan\left(\frac{\theta}{2}\right)}{1 + (y_0/x_0)\tan\left(\frac{\theta}{2}\right)}; \tag{3}$$

that is, the fringes bisect the angle of rotation.

When the images of this hologram are observed in the Fourier plane, it can be seen that the mentioned moiré fringes act as a low frequency linear grating producing two tiny bright spots, located symmetrically to the zero order, the separation d between which can be easily calculated from the given spacing Δ, as

$$d = \frac{\lambda D}{\Delta} = \frac{2D}{z}(x_0^2 + y_0^2)^{1/2}\sin\left(\frac{\theta}{2}\right),$$

where D is the distance between the hologram and the Fourier plane when the former is illuminated with a spherical convergent beam. That is, the separation between the spots is proportional to the magnitude of the displacement of the center of the Fresnel zones.

Returning to expression (1), it may be written as follows:

$$t \propto (2A_1^2 + 2A_2^2) + A_1A_2 \Bigg[\exp(i\phi) \exp\left\{\frac{i\pi}{\lambda z}[(x-x_0)^2+(y-y_0)^2]\right\} + \exp(i\phi')\exp\left\{\frac{i\pi}{\lambda z}\{[x-(x_0+\alpha)]^2+[y-(y_0+\beta)]^2\}\right\}\Bigg] + A_1A_2\Bigg[\exp(-i\phi)\exp\left\{-\frac{i\pi}{\lambda z}[(x-x_0)^2+(y-y_0)^2]\right\} + \exp(-i\phi')\exp\left\{-\frac{i\pi}{\lambda z}\{[x-(x_0+\alpha)]^2 + [y-(y_0+\beta)]^2\}\right\}\Bigg],$$

where $\alpha = x_0(\cos\theta - 1) + y_0 \sin\theta$, $\beta = y_0(\cos\theta - 1) - x_0 \sin\theta$, and ϕ' is a constant. The first term will give rise to the zero order, and the other two terms, to the plus-one and minus-one orders, respectively. Taking into account only the plus-one order, it may be expressed as

$$t^{(+1)} \propto A_1A_2 \exp(i\phi) \exp\left\{\frac{i\pi}{\lambda z}[(x-x_0)^2+(y-y_0)^2]\right\} * \{\delta(x)\delta(y) + \exp[i(\phi'-\phi)]\delta(x-\alpha)\delta(y-\beta)\},$$

where $*$ denotes a convolution product.

The corresponding intensity in the Fourier plane will be

$$I = C\left|1 + \exp\left\{-i\left[\frac{2\pi}{\lambda D}(x\alpha + y\beta) - (\phi'-\phi)\right]\right\}\right|^2 = 2C\left\{1 + \cos\left[\frac{2\pi}{\lambda D}(x\alpha + y\beta) - (\phi'-\phi)\right]\right\},$$

that is, a family of straight lines. It is easily shown that the slope of this family is also given by expression (3). The spacing Δ' between the straight lines results proportional to expression (2), that is,

$$\Delta' = \frac{\lambda D}{2(x_0^2 + y_0^2)^{1/2}\sin\left(\frac{\theta}{2}\right)} = \frac{D}{Z}\Delta.$$

Neither the pupil in the object plane during the exposures nor the one limiting the reconstruction process was taken into account in calculations, but their inclusion does not affect the final results substantially.

The experimental setup is shown in Fig. 1. A part of a collimated laser beam is focused on the object plane by a small lens, while the rest of the beam passes through pupil P. Both waves interfere on plate H, which is rotated between exposures. Figures 2 and 3 show the moiré fringes in the hologram and the intensity in the Fourier plane. Results confirm the described method. If two spherical wave fronts are used instead of one, the location of the axis of rotation can be determined.

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