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Unicycle Robot's Navigation Control with Obstacle Avoidance and Asymptotic Stability

[1]Juan Andrés Roteta Lannes and [2]Andres Gabriel Garcia

[1]*GIMAP-Scientific Research Commission of the Province of Buenos Aires (CIC), Faculty of Bahía Blanca Regional, National Technological University, Bahía Blanca, Buenos Aires, Argentina*
[2]*Department of Electrical Engineering, Faculty of Bahía Blanca Regional, National Technological University, Bahía Blanca, Buenos Aires, Argentina*

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Corresponding Author:
Juan Andrés Roteta Lannes
GIMAP-Scientific Research Commission of the Province of Buenos Aires (CIC), Faculty of Bahía Blanca Regional, National Technological University, Bahía Blanca, Buenos Aires, Argentina
Email: juanandresrl25@gmail.com

Abstract: This study builds upon the groundbreaking research of Asymptotic stability of unicycle-like robots with the Bessel's controller continuing the exploration of asymptotic stability for non-holonomic robots through kinematic modeling that allows for obstacle avoidance. utilizing the previously derived Bessel's controller, the study defines an avoidance region containing obstacles, presenting an algorithm that relies solely on the distance to the obstacle. This novel algorithm introduces a new set of Ordinary Differential Equations (ODEs) to recalibrate the controller. A MATLAB/Simulink example demonstrates the exact algorithm using Bessel's functions and an approximate solution, emphasizing a more tractable hardware implementation. The paper contributes a significant advancement in the field, combining asymptotic stability, obstacle avoidance, and efficient hardware implementation. In conclusion, this study introduces and validates a pioneering navigation algorithm tailored for unicycle-like robots, ensuring asymptotic stability even in the presence of obstacles. Building upon the earlier research framework utilizing Bessel's controllers, the paper highlights instances of asymptotic stability and convergence near the origin, addressing a notable gap in the existing literature regarding path planning and navigation algorithms for obstacle avoidance with asymptotic stability. The research trajectory initiated by previous paper proves instrumental in advancing the understanding and practical implementation of stable navigation algorithms for robotic systems, particularly in scenarios involving obstacles. This study not only extends the achievements of the previous work but also provides valuable insights and recommendations for future research directions in the pursuit of robust and efficient robotic navigation.

Keywords: Kinematic Model, Nonholonomic Dynamics, Obstacle Avoidance, Hybrid Systems

Introduction

Modeling mechanical systems that roll without slipping presents various challenges, depending on whether a dynamic or kinematic approach is employed (Siciliano and Khatib, 2008; Angeles and Kecskemethy, 1995). This study primarily focuses on dynamic models, which account for all possible physical interactions, resulting in a set of Ordinary Differential Equations (ODEs) corresponding to the degrees of freedom of the mechanical system (Angeles and Kecskemethy, 1995).

Dealing with a large number of ODEs and establishing a control law for asymptotic stability remain significant challenges (Kostić *et al*., 2009; Udwadia and Kalaba, 1992; Skowronski, 2012; García, 2020). Drawing on the advancements of (García, 2020) in achieving asymptotic stability for unicycle-like robots using Bessel's controller, our work extends this achievement to address the inclusion of obstacles for obstacle avoidance, ensuring asymptotic stability.

In this context, there is a growing interest in controlling more tractable kinematic models for mobile robots, particularly within the subset of wheel planar kinematic models (Siegwart *et al.*, 2011; Ostrowski, 1999; Lecanda and Fernandez, 2008). Unicycle-like robots, as universal models for a wide range of wheeled robots (Garcia and Agamennoni, 2012), offer a practical framework for navigation and control. According to Murray and Sastry (1993), any nonholonomic system can be written in a universal chain form, allowing the use of unicycle models as general non-holonomic dynamics.

Safe mobile robot navigation necessitates effective obstacle-avoidance strategies, as highlighted by Dang and Bui (2023). While various methods like fuzzy logic, neural networks, and potential fields generate clear paths, none guarantee asymptotic stability (Shitsukane *et al.*, 2018a-b). Building upon García (2020), this study introduces an extension that incorporates obstacle avoidance with an exclusion region, ensuring asymptotic stability.

The structure of this study unfolds as follows: We begin with the introduction of necessary definitions and notations, followed by the presentation of the unicycle robot with Bessel's controller and closed-form trajectory solutions in section 'unicycle robot with Bessel's controller.' The subsequent sections delve into obstacle avoidance constraints, main theorems, and corollaries regarding asymptotic stability and obstacle avoidance, with practical simulation examples using MATLAB in section 'examples.' The conclusion offers insights and outlines avenues for future research.

Notation and Definitions: Preliminaries

In this section, some definitions are provided for the sake of completeness.

Derivatives with Respect to Time and Differential

Time derivatives are indicated as:

$$\dot{x}(t) = \frac{dx(t)}{dt}$$

where, *dt* means a time's differential.

Closed-form Controller

According to García (2020), an asymptotic stabilizing controller for unicycle robots is out of Bessel's functions. This controller leads the following notation:

$$\begin{aligned} &X(t) = G(\theta).\bar{C} \\ &G(\theta) = R(\theta)' \cdot \sum_{i=1}^{N} C_i \cdot L(\theta) \\ &L(\theta) = \begin{bmatrix} J_i(\theta) & \cdot \theta^{i+1} \\ -J_{i+1}(\theta) \cdot \theta^{i+1} \end{bmatrix} \\ &i = 1, \cdots, N; \forall N \in \mathbb{R} \end{aligned} \tag{1}$$

Defined C as:

$$\bar{C} = \begin{bmatrix} C_1 \\ C_2 \\ \cdot \\ \cdot \\ \cdot \\ C_n \end{bmatrix} = L^{-1}(\theta(0)) \cdot R(\theta(0)) \cdot \begin{bmatrix} x(0) \\ y(0) \end{bmatrix}$$

With:

$$R(\theta) = \begin{bmatrix} \cos(\theta)\sin(\theta) \\ -\sin(\theta)\cos(\theta) \end{bmatrix}$$

$G(\theta)$ is considered the Bessel control and the dynamics of the unicycle robot is one system. Where $X(t)$ is the robot posicion in instant *t*.

U*nicycle Robot with Bessel's Controller*

In this study, unicycle robots are considered (Fig. 1) and (Eq. 2):

$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} \cos(\theta) & 0 \\ \sin(\theta) & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} \tag{2}$$

where, the control inputs $(u_1, u_2) \in \mathfrak{R}^1 \times \mathfrak{R}^1$.

According to García (2020), asymptotic stability can be achieved for this kind of robot globally using the following controller:

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} H(\theta) \cdot C \\ a \cdot \theta \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{N} (2 \cdot i + 1) \cdot a \cdot C_i \cdot J_i(\theta) \cdot \theta^{i+1} \\ a \cdot \theta \end{bmatrix}, \tag{3}$$

$\forall N \in \mathbb{N}, \forall a < 0$

$H(\theta)$ is considered the Bessel control without the dynamics of the unicycle robot. Where u_1 and u_1 are the control inputs to said robot.

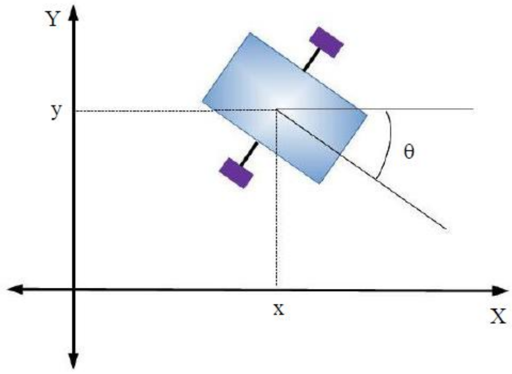

Fig. 1: Unicycle-like robot with coordinates

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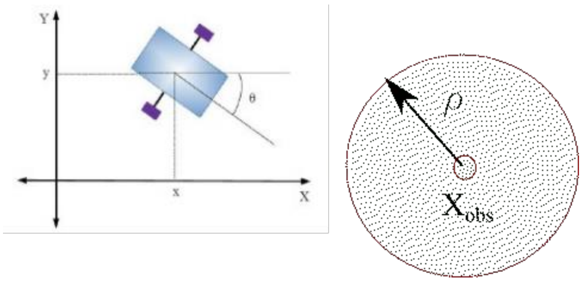

Fig. 2: Unicycle-like robot with obstacle avoidance region

Obstacle Avoidance Constraint

Considering the difference between the location of the robot and the obstacle $X_{obstacle}$, that is, its distance, and assuming that this must be greater than the radius (δ) of the obstacle (Fig. 2).

In this way, the following general obstacle avoidance constraint is defined:

$$\varphi(X, X_{obstacle}) \leq 0 \tag{4}$$

where:

$$X = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}, \quad X_{obstacle} = \begin{bmatrix} x_{obstacle} \\ y_{obstacle} \end{bmatrix}$$

The next section provides an asymptotic stability result as an extension of the theorem presented in García (2020) for a unicycle with obstacle avoidance.

Results

Considering the constraint (4), it is possible to conclude two possible scenarios:

- $\varphi(X, X_{obstacle}) < 0$, then the obstacle is far away and the constraint inactive
- $\varphi(X, X_{obstacle}) = 0$, then the border is reached and an equality constraint is added to the control system

As can be seen from previous notations, the constant matrix C can be chosen with an arbitrary number of rows, so this flexibility will be exploited in what follows to avoid obstacles with an asymptotic stability guarantee.

Theorem 1 (Obstacle Avoidance Controller)

Given a unicycle robot (2) with an obstacle avoidance region (4) and the controller (3) with the following change's direction law:

$$\begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix} = \begin{bmatrix} H(\theta(t)) \cdot C(k) \\ a \cdot \theta(t) \end{bmatrix}, \quad \forall N \in \mathbb{N}, \forall a < 0$$

$$\begin{cases} G(\theta(T)) \cdot C(k+1) = G(\theta(T)) \cdot C(k) \\ H(\theta(T)) \cdot C(k+1) = \beta \cdot H(\theta(T)) \cdot C(k) \end{cases} \tag{5}$$

$$\beta < 0, \beta \in \Re$$

where, $\{u_1(k) = u_1(T-dt), u_1(k+1) = u_1(T+dt)\}$ with T the instant of time the avoidance region is reached: $\varphi(X, X_{obstacle}) = 0$ and $\theta(T)$ is the attitude when reaching that boundary.

This control law provides asymptotic for any given initial condition:

$$\begin{cases} X(t) \to 0, & \text{Finite swtinchihg's number} \\ X(t) \to \text{constant}, & \text{Infinite switching's number} \end{cases}$$

$$X(0) = X_0, \quad \theta(0) = \theta_0$$
$$X_0 = G(\theta_0) \cdot C(0)$$

Proof

Observing that reaching the boundary avoidance region: $\varphi(X, X_{obstacle}) = 0$ means that the robot must necessarily change its forward speed direction u_1:

$$\frac{u_1(k+1)}{u_1(k)} = \beta < 0$$

where, $\{u_1(k) = u_1(T-dt), u_1(k+1) = u_1(T+dt)\}$ with T the instant of time the avoidance region is reached:

$$\varphi(X, X_{obstacle}) = 0$$

In this way and taking into account the trajectories' continuity, the following set of equations can be written:

$$\begin{cases} G(\theta(T)) \cdot C(k+1) = G(\theta(T)) \cdot C(k) & \text{Continuity} \\ H(\theta(T)) \cdot C(k+1) = \beta \cdot H(\theta(T)) \cdot C(k) & \text{Direction} \end{cases} \tag{6}$$

On the other hand, every time the control input u_1 is changed, a new trajectory starts, and the given impact point on the boundary region can be considered as a new initial point, considering as a constant vector in each trajectory's piece, two cases must be considered.

Finite Switching's Number

In this case, each trajectory piece can be regarded as a new initial point with asymptotic stability to the origin guarantee (provided by the Bessel's controller), so after a finite number of switches, the origin is reached, as long as the origin is outside the obstacle avoidance region.

Infinite Switching's Number

This case could be the case for the origin inside the obstacle's avoidance region or far away from the trajectory where the obstacle avoids a finite amount of switching numbers.

In this case, the switching law at (6) can be analyzed when the asymptotic behavior of $\theta(t)$ is, in fact, near to zero:

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$$\begin{bmatrix} G(\theta(T)) \\ H(\theta(T)) \end{bmatrix} \cdot C(k+1) = \begin{bmatrix} G(\theta(T)) \\ \beta \cdot H(\theta(T)) \end{bmatrix} \cdot C(k)$$

Then, the asymptotic formulas for small arguments (see for instance "special functions for engineers and applied mathematicians", provides:

$$\begin{bmatrix} G(\theta(T)) \\ H(\theta(T)) \end{bmatrix} \approx \begin{bmatrix} \frac{\theta^3}{2} & \frac{\theta^5}{8} & \frac{\theta^7}{48} \\ -\frac{\theta^4}{8} & -\frac{\theta^6}{48} & -\frac{\theta^8}{384} \\ 3 \cdot a \cdot \frac{\theta^3}{2} & 5 \cdot a \cdot \frac{\theta^5}{8} & 7 \cdot a \cdot \frac{\theta^7}{48} \end{bmatrix} \quad (\theta(t) \to 0)$$

The inverse can be also asymptotically computed:

$$\begin{bmatrix} G(\theta(T)) \\ H(\theta(T)) \end{bmatrix}^{-1} \approx \begin{bmatrix} -\frac{13}{\theta^3} & -\frac{48}{\theta^4} & \frac{1}{a \cdot \theta^3} \\ \frac{132}{\theta^5} & \frac{384}{\theta^6} & -\frac{12}{a \cdot \theta^5} \\ -\frac{432}{\theta^7} & -\frac{1152}{\theta^8} & \frac{48}{a \cdot \theta^7} \end{bmatrix} \quad (\theta(t) \to 0)$$

Finally, it is possible to get an asymptotic expansion for the equilibrium reached:

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} G(\theta(T)) \\ H(\theta(T)) \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \beta \end{bmatrix} \cdot \begin{bmatrix} G(\theta(T)) \\ H(\theta(T)) \end{bmatrix} \approx \begin{bmatrix} x(t) \cdot b \\ y(t) \end{bmatrix} \quad (\theta(t) \to 0)$$

With $b \neq 0$. This completes the proof.

Corollary 1

The controller (5) has a solution for any impact point, that is, the ODE that governs the system will always have a solution.

Proof

Taking into account the definitions at (1-3): $G(\theta), H(\theta)$, it is clear that the rank of the matrix:

$$\begin{bmatrix} G(\theta(t)) \in \Re^N \\ H(\theta(t)) \in \Re \end{bmatrix}$$

This matrix possesses a rank equal to 3 for any number N. Then a solution to (6) arises:

$$C(k+1) = \begin{bmatrix} G(\theta(T)) \\ H(\theta(T)) \end{bmatrix}^{+} \cdot \begin{bmatrix} G(\theta(T)) \\ \beta \cdot H(\theta(T)) \end{bmatrix} \cdot C(k)$$

where, $(.)^+$ means the pseudo inverse or Moore-Penrose inverse (Ben-Israel, 1980). This completes the proof.

Sometimes, it is very practical to use polynomial approximations instead of Bessel's functions.

Corollary 2 (Asymptotic Approximation)

The controller (3) can be approximated with the following polynomial simplification for small angle attitudes θ:

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} H(\theta) \cdot C \\ a \cdot \theta \end{bmatrix} \simeq \begin{bmatrix} \sum_{i=1}^{N} (2 \cdot i + 1) \cdot a \cdot C_i \cdot \frac{\left(\frac{\theta}{2}\right)^i}{n!} \cdot \theta^{i+1} \\ a \cdot \theta \end{bmatrix},$$

$$\forall N \in \mathbb{N}, \forall a < 0, 0 < \theta \ll \sqrt{2}$$

Proof

Utilizing the asymptotic formulas for small arguments for instance "special functions for engineers and applied mathematicians":

$$J_i(x) \simeq \frac{\left(\frac{x}{2}\right)^i}{i!}, \quad i \in \mathbb{N}, \quad 0 < x \ll \sqrt{i+1}$$

where, $i!$ stands for the classical factorial function. This completes the proof.

Simulation Example

Figure 3, using the robotics toolbox in Simulink with the following parameters:

Wheel Diameter = 0.105 m, Robot's Width = 0.015 m

Setting an initial condition $X(0) = [1.5 \quad -5]$, $\theta(0) = 2$ and with $a = -0.0001$, $\beta = -5$, the results in Fig. 4 were obtained, using the asymptotic approximation given by corollary 2 and Fig. 5 for a comparison with the exact controller form in (2).

It is clear that the robot reverts the forward velocity as soon as the avoidance region is reached and then a new trajectory is scheduled to reach the origin.

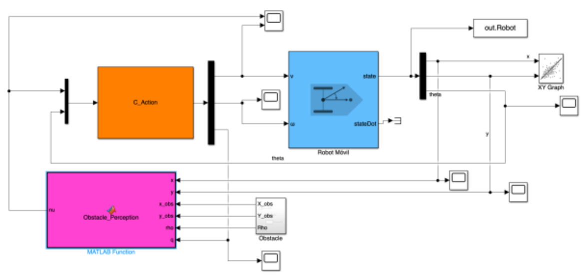

Fig. 3: Simulink's block diagram

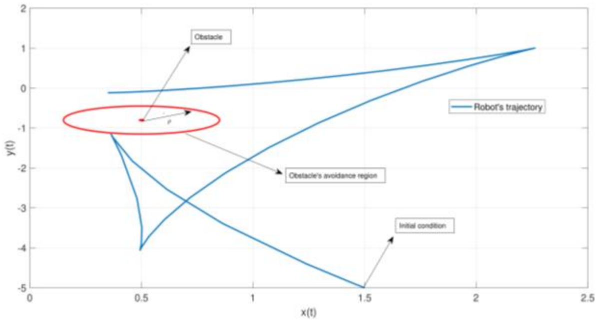

Fig. 4: Robot's trajectory avoiding the obstacles' region using asymptotic Bessel's approximations

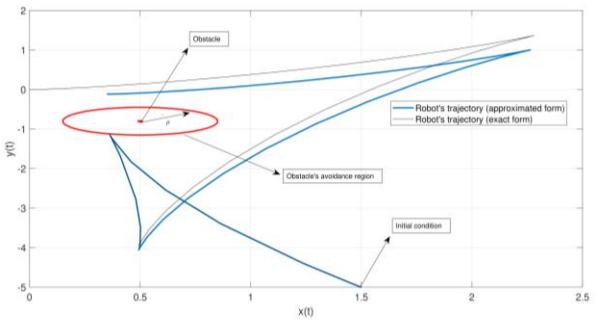

Fig. 5: Robot's trajectory avoiding the obstacles' region using Bessel's exact form

Discussion

Avoiding obstacles while keeping asymptotic stability can be difficult without pre-planned trajectories or ad-hoc algorithms without a convergence guarantee (Kostić *et al*., 2009; Udwadia and Kalaba, 1992; Skowronski, 2012; García, 2020).

In this study, utilizing the fact that our previous Bessel's controller is asymptotically stable for any number of terms in matrix *C* (García, 2020), a set of algebraic equations was derived to change the lineal velocity direction when the robot hits defined regions surrounding an obstacle to avoid.

There is no doubt that, in addition to providing asymptotic stability for avoiding obstacles without pre-planned routes or prior knowledge of the location of obstacles (on-line algorithm), a simple sensor to measure the distance to obstacles along a running trajectory is required (for example, low cost ultrasonic or optical distance sensor). As a result, the algorithm is efficient and computationally tractable for any robot with even low computational resources.

Conclusion

In this study, we have introduced and demonstrated a novel navigation algorithm with asymptotic stability tailored for unicycle-like robots navigating through environments with obstacles. Building upon the framework of Bessel's controllers, our previous work showcased instances of asymptotic stability and, in many cases, convergence to a region proximal to the origin at the periphery of obstacles.

Despite the extensive body of literature available, the scarcity or absence of path planning and navigation algorithms ensuring asymptotic stability while navigating around obstacles using distance-based strategies is noteworthy.

Potential avenues for future research include:

- Expanding the algorithm's capabilities to steer vehicles and more general robots, enabling effective evasion of moving obstacles
- Integration of non-linear noisy observers to reconstruct and filter noisy posture angle measurements, enhancing the robustness of the system
- Enhancement of asymptotic stability conditions, providing both necessary and sufficient criteria for scenarios both inside and outside the obstacle's region when approaching the goal
- Incorporation of dynamic models into optimal control strategies, offering a more comprehensive understanding of the system's behavior

This revised conclusion aims to enhance clarity, coherence, and emphasis on the significance of the presented algorithm and the potential directions for future research.

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Author's Contributions

Juan Andrés Roteta Lannes: Performed numerical analysis and modeling with MATLAB. Participated in writing and revising the manuscript.

Andrés Gabriel García: Developed the research design and coordinated the study. Also participated in writing and revising the manuscript.

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Ethics

This article is original and contains unpublished material. The corresponding author confirms that all of the other authors have read and approved the manuscript and that no ethical issues are involved.

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