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Spatial partial coherence in Young's interferograms

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Abstract

Young's interferograms with high visibility reveals a high degree of spatial coherence of first order. But, spatially partial coherence of second order can be observed when it interferes itself through a compensated Michelson's interferometer attached at the exit of the Young's slit pair. We show that the patterns at the exit of the Michelson's interferometer are Young's interferograms with modulation fringes, which allow an estimation of the degree of the high order spatial coherence. © 1998 Elsevier Science Ltd. All rights reserved.

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1. Fundamentals

Fig. 1 shows the experimental setup. A pair of Young's slits, of width a along the x-axis, length $l \gg a$ along the y-axis and separation $b > a$, is illuminated by an He–Ne laser. The mirrors M1 and M2 of a compensated Michelson's interferometer reflect the Young's interference pattern it produces. The reflected patterns will be superimposed at the CCD sensor, which is attached at the exit of the Michelson's interferometer.

The reflected patterns can interfere depending on the tilting angle ϕ, which introduces a relative lateral shift between the patterns. This interference allows us to analyse the coherence patch of the optical field [1]. Indeed, it will produce a modulation of the Young's pattern whose fringe orientation, period and visibility depend on this lateral shift and on the structure of the coherence patch of the optical field.

The intensity distribution of the interferogram at the exit and of a Michelson's interferometer when the optical field is spatially partial coherent [1] is given by

$$I(x,y) = I_1(x,y) + I_2(x - X, y - Y)$$

$$+ 2W_{12}^{(r)}(x,y;X,Y). \tag{1}$$

$I_j(x,y)$ is the intensity distribution of the pattern from the mirror j ($j = 1, 2$); (X,Y) is the relative lateral shift between the patterns reflected by the two mirrors and $W_{12}^{(r)}(x,y;X,Y) = \mathrm{Re}|\mu(x - X, y - Y)E_1^*(x,y)E_2(x - X, y - Y)|$, where Re denotes the real part; $\mu(x - X, y - Y) = |\mu(x - X, y - Y)|e^{i\alpha_{XY}}$ is the complex degree of spatial coherence of the optical field, with $0 \le |\mu(x - X, y - Y)| \le 1$, $\mu(x,y) \equiv 1$ and $\alpha_{XY} = \alpha_{XY}(x - X, y - Y)$; $E_j(x,y)$ is the amplitude distribution of the pattern from the mirror j, and the asterisk denotes complex conjugate.

So, the visibility of such an interferogram [1] will be given by $V = [2\sqrt{I_1(x,y)}\sqrt{I_2(x - X, y - Y)}]/[I_1(x,y) + I_2(x - X, y - Y)]|\mu(x - X, y - Y)|$. Thus, for $I_1(x,y) \approx I_2(x - X, y - Y)$, V will be large inside the coherence patch of the optical field and will be small or zero outside of it.

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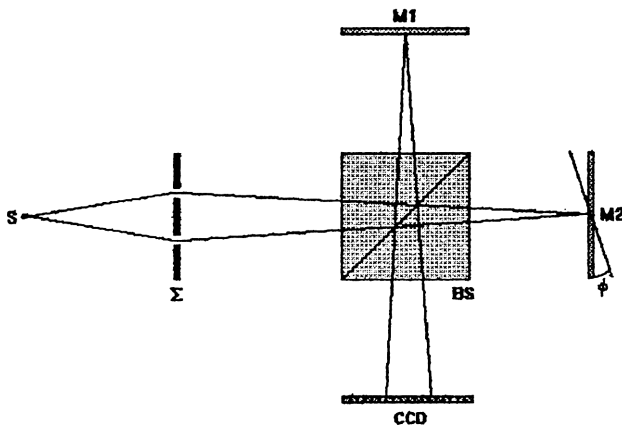[2] postmaster@ciop.edu.ar

Fig. 1. Experimental setup to obtain an interference pattern from a Young's interferogram.

According to the experimental setup, let us assume that $I_f(x,y)$ represents a Young's pattern of high visibility, which is due to a high degree of first order spatial coherence of the optical field. Therefore, the visibility of the modulation at the exit of the Michelson's interferometer will reveal a spatially partial coherence of second order, due the partial correlation between pairs of corresponding radiators at the interferometer mirrors M1 and M2.
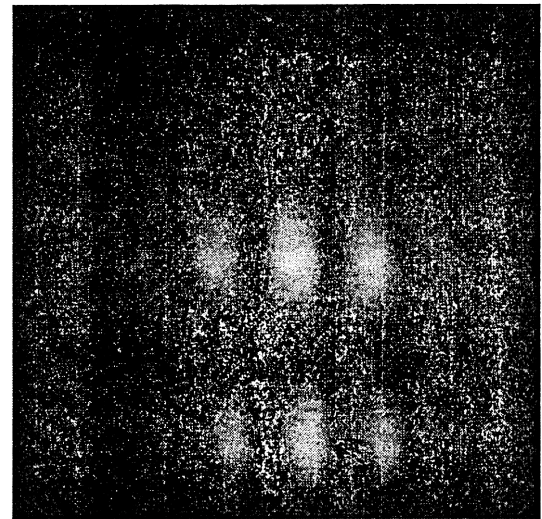
Taking into account that $k = 2\pi/\lambda$, with λ the wavelength and L the optical path length into the compensated Michelson's interferometer [2], we have

$$I_1(x,y) = |E_0|^2 \left(\frac{\sin\left(\frac{ka}{L}x\right)}{\frac{ka}{L}x} \right)^2$$

$$\times \left(\frac{\sin\left(\frac{kl}{L}y\right)}{\frac{kl}{L}y} \right)^2 \cos^2\left(\frac{kb}{L}x\right), \tag{2a}$$
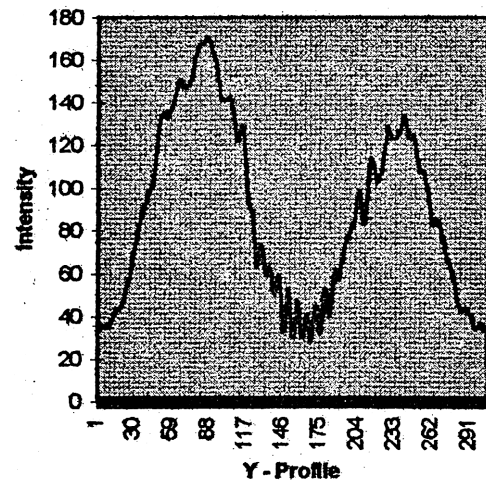
$$I_2(x - X, y - Y) = |E_0|^2 \left(\frac{\sin\left(\frac{ka}{L}(x - X)\right)}{\frac{ka}{L}(x - X)} \right)^2$$

$$\times \left(\frac{\sin\left(\frac{kl}{L}(y - Y)\right)}{\frac{kl}{L}(y - Y)} \right)^2 \cos^2\left(\frac{kl}{L}(x - X)\right), \tag{2b}$$

$$W_{12}^{(r)}(x,y;X,Y) = |E_0|^2|\mu(x - X, y - Y)|\cos \alpha_{XY}$$

$$\times \left(\frac{\sin\left(\frac{ka}{L}(x - X)\right)}{\frac{ka}{L}(x - X)} \right) \times \left(\frac{\sin\left(\frac{ka}{L}x\right)}{\frac{ka}{L}x} \right)$$

$$\times \left(\frac{\sin\left(\frac{kl}{L}(y - Y)\right)}{\frac{kl}{L}(y - Y)} \right) \cos\left(\frac{kb}{L}(x - X)\right)$$

$$\times \cos\left(\frac{kb}{L}x\right). \tag{2c}$$

For $X = Y = 0$ Eqs. (1) and (2a)–(c) yield $I(x,y) = 4 | E_0 |^2\{\sin[(ka/L)x]/(ka/L)x\}^2\{\sin[(kl/L)y]/$
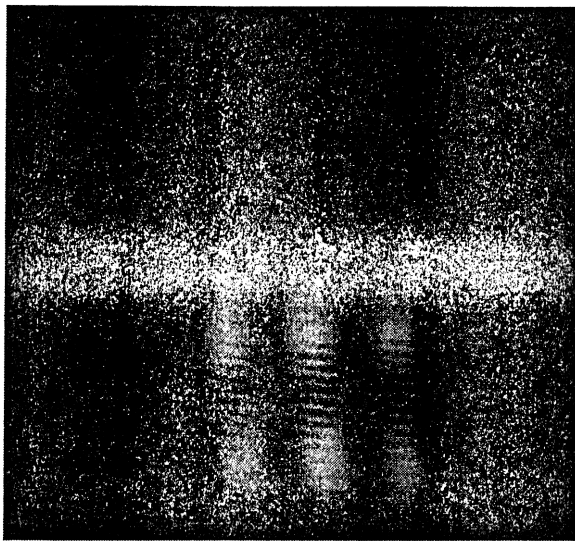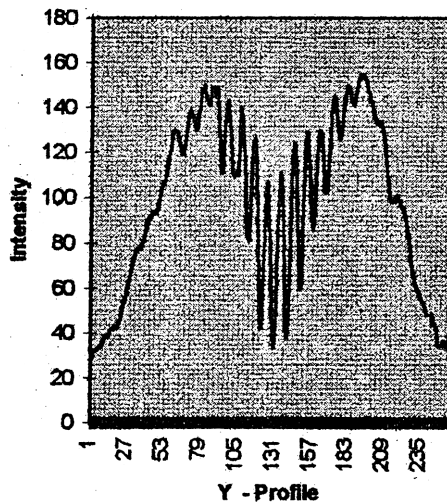

a

b

Fig. 2. (a) Young's interferograms outside of the coherence patch of the optical field. (b) A vertical profile in (a).
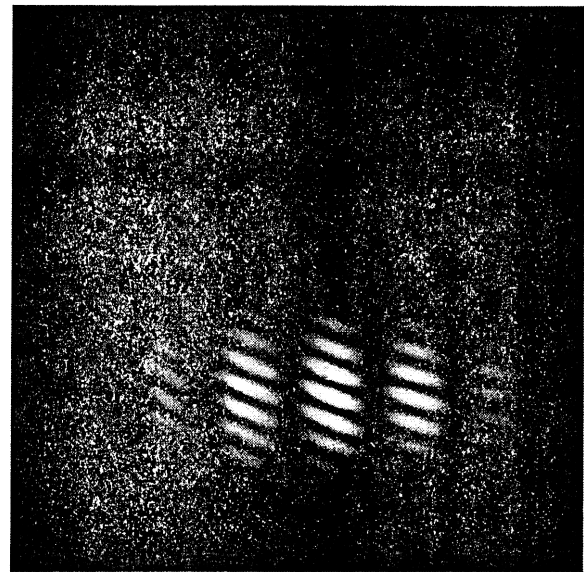
a

b

Fig. 3. (a) Young's interferograms partially located inside of the coherence patch of the optical field. (b) A vertical profile in (a).

$(kl/L)y\}^2 \cos^2[(kb/L)x]$, which exhibits the same structure as an individual Young pattern without modulation, and allows us to calibrate the Michelson interferometer. The modulation fringes on the Young pattern will be mainly described by the first cosine term in parentheses in expression (3), which is obtained from Eq. (2c), by applying the trigonometric identity for the product of cosine functions [1–3], i.e.

$$\frac{1}{2}|\mu(x - X, y - Y)|\cos \alpha_{XY}\left\{\cos\left[\frac{2kb}{L}\left(x - \frac{X}{2}\right)\right]\right.$$

$$\left. + \cos\left(\frac{kb}{L}X\right)\right\}.$$ (3)

If $X < (\lambda L/b)$ expression (3) approaches to $|\mu(x - X, y - Y)|\cos \alpha_{XY}\{\cos^2[(kb/L)x] + (kb/2L)X \sin[(2kb/L)x]\}$. The coefficient of the second term into the parenthesis will be smaller than 1, so that the sinus function can be considered as a perturbation of the cosine squared. In other words, the profile of the modulation fringes (when they appear) will be essentially the cosine squared function.

a
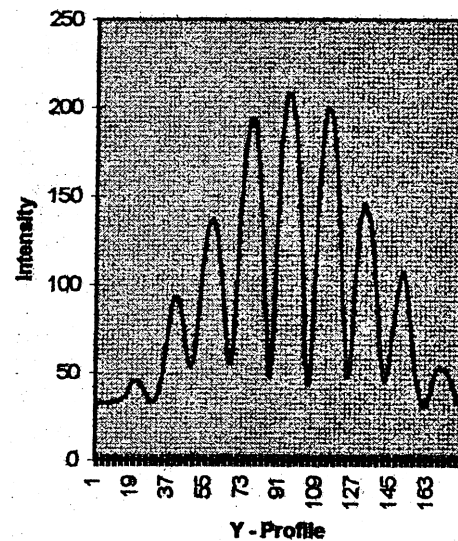
b

Fig. 4. (a) Superimposed Young's interferograms, completely located inside of the coherence patch of the optical field. (b) A vertical profile in (a).

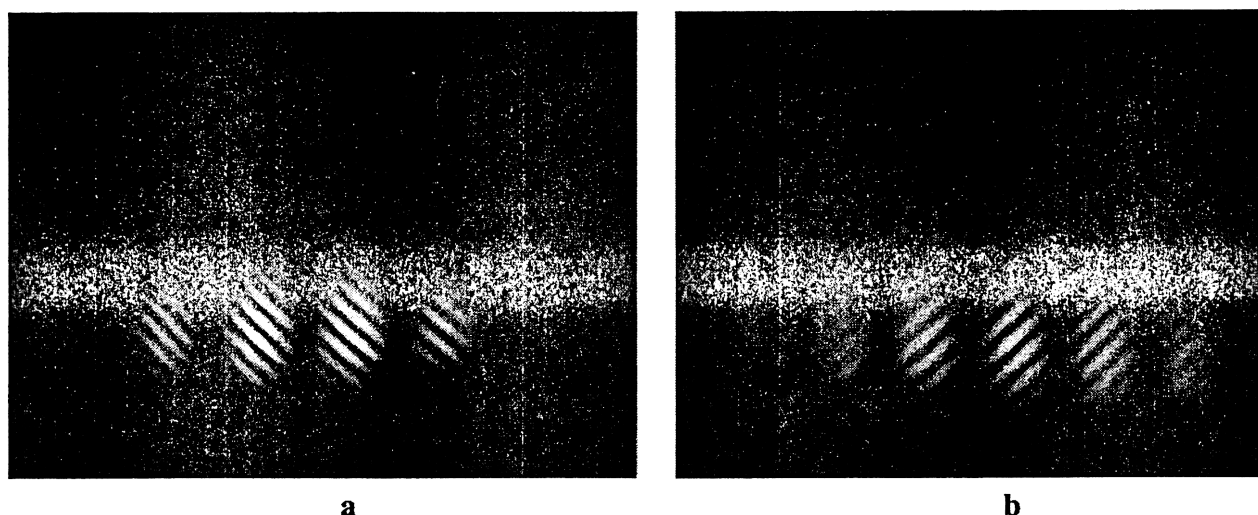a   b

Fig. 5. Superimposed Young's interferograms with (a) left and (b) right horizontal shifts. The histograms of their vertical profiles are similar as in Fig. 4b.

2. Experimental results

Figures 2–5 show the portion of the diffraction central peak of the Young's patterns at the exit of the Michelson's interferometer. The size of the coherence patch of the optical field is of the order of the area of those diffraction peaks, i.e. $|\mu(x - X, y - Y)| \rightarrow 1$ inside them.

In Fig. 2a the Y shift is greater than the size of the coherence patch along this axis. As a consequence, $|\mu(x - X, y - Y)| \rightarrow 0$. Indeed, there is a low intensity modulation between the superimposed patterns and an absence of modulation into the Young's lobes (Fig. 2b).

The square cosine-like modulation in this region increases significantly when the Y shift becomes smaller, but the lack of visibility is even apparent in the Young's lobes (Fig. 3a and b).

The high visibility of the square cosine-like modulation, i.e. $|\mu(x - X, y - Y)| \rightarrow 1$, will be apparent inside the whole Young's lobes as $Y \rightarrow 0$ (Fig. 4a and b).

In Fig. 5a and b the Y shift is negligible but not the X shift. After a shift of about two orders of the Young's pattern, i.e. $X \approx 2\lambda L/b$, the expression (3) approaches to $\cos^2([kb/L]x + \alpha_{XY})$. Thus, the modulation fringes are of high visibility (square cosine-like) and covers the whole Young's lobes. It is also interesting to observe the change in the orientation of the modulation fringes, which is essentially related to the

change of sign in the coherence phase α_{XY} when the pattern shift goes from the left to the right.

3. Conclusions

Spatial coherence of second order can be analysed if a Young's pattern interferes itself through a compensated Michelson interferometer. Modulation fringes on the Young's pattern can be observed at the exit of this device, whose visibility provides an estimation of the degree of the high order spatial coherence.

Acknowledgements

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